HW. # 16

Homework problems are taken from several textbooks. The problems are color coded to indicate level of difficulty. The color green indicates an elementary problem, which you should be able to solve effortlessly. Yellow means that the problem is somewhat harder. Red indicates that the problem is hard. You should attempt the hard problems especially.

Evaluate the double integrals by converting to polar coordinates..

1. $\iint_B \frac{y}{x}$; B is the region in the first quadrant bounded by the lines y = 0, y = x, and the circles $x^2 + y^2 = 2$ and $x^2 + y^2 = 25$

2. $\iint_B (x^2 + y^2)^{3/2}$; B is the half-disc $x^2 + y^2 \le 1$, $x \le 0$.

3. $\iint_B \frac{1}{\sqrt{x^2 + y^2}}$; B is the annular region given by $4 \le x^2 + y^2 \le 9$.

4. $\iint_B (x^2 + y^2)$; B is the disc $x^2 + y^2 \le 4x$.

5. $\iint_B \tan^{-1}\left(\frac{y}{x}\right)$; B is the sector in the first quadrant between the circles $x^2 + y^2 = \frac{1}{4}$ and $x^2 + y^2 = 1$ and the lines $y = x/\sqrt{3}$ and y = x.

6. $\int_0^1 \int_0^{\sqrt{1-x^2}} x^2 y \, dy \, dx$

7. $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} e^{-x^2-y^2} \, dx \, dy$

8. $\int_0^4 \int_x^4 (x^2 + y^2)^{3/2} \, dy \, dx$

9. $\int_1^2 \int_{-y}^{y/\sqrt{3}} \frac{1+\sqrt{x^2+y^2}}{\sqrt{x^2+y^2}}\,dxdy$

Evaluate the double integral by making an appropriate change of variables.

10. $\iint_B (x+y)^2$; B is the region bounded by the lines x + y = 0, x + y = 1, 2x – y = 0, and 2x –y = 1.

11. $\iint_B \frac{1}{y}$; B is the region bounded by $y^3 = x^2$, $y^3 = 6x^2$, y = 2x, and y = 3x.

12. $\iint_B \frac{x^2 \sin xy}{y}$; B is bounded by $x^2 = \pi y/2$, $x^2 = \pi y$, $y^2 = x/2$, and $y^2 = x$.

13. $\iint_B x^2$; B is bounded by y = x, y = 3x, y = -1 – x, and y = -3 – x.

14. $\iint_B \frac{9x^2+8y^2}{xy}$; B is the region in the first quadrant bounded by the ellipses $x^2/4 + y^2/9 = 1$ and $x^2/16 + y^2/36 = 1$ and the parabolas $y = x^2/2$ and $y = 2x^2$

Evaluate the following double integrals over unbounded regions.

15. $\iint_B e^{-x^2-y^2}$; B is the entire xy-plane.

16. $\iint_B \frac{1}{1+(x^2+y^2)^2}$; B is the portion of the first quadrant bounded by the x-axis and the line y = x.

17. $\iint_B \frac{1}{(x^2+y^2)^{3/2}}$; B is the half-plane to the right of the vertical line x = 1.